

SPOTLIGHT PROBLEMS – WEEK 1

- (1) The graph of Fahrenheit temperature, °F, as a function of Celsius temperature, °C, is a line. You know that 212°F and 100°C both represent the temperature at which water boils. Similarly, 32°F and 0°C both represent water's freezing point.
 - (a) What is the slope of the graph?
 - (b) What is the equation of the line? (Write two equations – One expressing degrees Celsius as a function of degrees Fahrenheit, and then one expressing degrees Fahrenheit as a function of degrees Celsius.)
 - (c) Use the equation to find what Fahrenheit temperature corresponds to 20°C.
 - (d) What temperature is the same number of degrees in both Celsius and Fahrenheit?
- (2) While $f \circ g \neq g \circ f$ in general, provide an example of two different functions $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ such that $f \circ g = g \circ f$. Next, give an *infinite* set of functions (not all constants) where this holds true for any two functions within the set. (Hint: use the fact that addition and multiplication are commutative.)
- (3) Suppose that 100 kilograms of a particular isotope of plutonium (^{239}Pu) is deposited at a nuclear waste site. The half-life of plutonium-239 is 24,100 years.
 - (a) How much will remain after 100,000 years?
 - (b) After 150,000 years?
 - (c) When will only 50 grams remain?
- (4) Determine whether the following statements are true or false. If true, prove it. Otherwise, give a counter example.
 - (a) Every strictly increasing function is invertible.
 - (b) If a strictly increasing function has an inverse, then its inverse is also increasing.
 - (c) Every polynomial function of odd degree is one-to-one.
 - (d) There is some polynomial function of odd degree that is one-to-one.